A Consistency-driven Approach to the CD-ROM Selection[1]

Zbigniew Duszak and Waldemar W. Koczkodaj

Zbigniew Duszak, Associate Researcher, Laurentian University, Department of Mathematics and Computer Science, Sudbury, Ontario, Canada P3E 2C6

Waldemar W. Koczkodaj, Associate Professor, Laurentian University, Department of Mathematics and Computer Science, Sudbury, Ontario, Canada P3E 2C6

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Abstract

The growing importance of CD-ROM information is becoming clear to professionals in the field of library science. It is not feasible for most university libraries to acquire all CD-ROM publications therefore a selection process must take place.

There are various ways of formulating priorities: trade-off methods, ratings, rankings, verbal statements and pairwise comparisons. Weights, reflecting the relative importance of the objectives concerned are a very common and valuable piece of information for achieving selections from a large population. The ranking method, according to Voogd [Voogd, 1983], is preferable by the majority of interviewed experts. However, in this case, there is no tool for validation of the knowledge. In practice, it is necessary to use a method that enables the decision makers to express their priorities in a more refined way. Therefore, we propose the pairwise comparison method for knowledge acquisition with the consistency measure [Koczkodaj, 1993] as a validation technique. Our experience shows, that this approach, called a consistency driven knowledge acquisition, supported by a properly designed computer software, greatly improves the problem of understanding and the quality of the selection process.

Why Do We Need A Better CD-ROM Tool?

A university library is committed to providing information services of the highest quality to its academic community. The modern library now requires the establishment and maintenance of a CD-ROM collection and network to provide its users with research services of the highest character. Evaluation and selection of CD-ROM's should be based on well-established guidelines for book and paper collections. CD-ROMs considered for acquisition should directly support the academic activities of the university. There are, however, new principles applicable only to CD-ROM technology. For example, a CD-ROM product should not be purchased in lieu of a corresponding print product simply because the CD-ROM product exists. Computer network compatibility is another example of exclusive applicability to CD-ROM. Some of the other characteristics have application to both paper and CD-ROM publications (e.g. price and contents), however they may have a different importance. It is fair to assume that CD-ROM publications are still more expensive than paper publications. The price of some CD-ROM publications are still in excess of the price of the entire Encyclopedia Britannica. On the other hand, one CD-ROM may accommodate 30,000 mid-size journal publications (containing text only). This figure may not look substantial, however, one must realize that it is equivalent to 50 volumes (or 50 years of publication!) of a journal with four annual issues containing 10 papers, each 15 pages long. The impact of one CD-ROM purchase to academic activities is more far-reaching than that of a single book or a journal.

In a good selection model, consideration should be given to the accuracy and authority of the CD-ROM contents, to ease of use, to the population size of potential users, and to the potential increase in usage that a CD-ROM might encourage over its established print counterpart. Where subscription prices are comparable among CD-ROM publishers, preference should be given to the publisher whose products are currently in use in the library. Annual subscription costs, networking costs, and hardware costs are also important aspects to be considered in the selection process. If the cost of maintaining a subscription to a given CD-ROM product outweighs its potential usage, consideration should be given, where applicable, to the online access through commercial databanks as an alternative option. CD-ROMs should be acquired in the following priority: periodical indexes and abstracts, collection development tools, full texts (journals, newspapers), reference material (directories, dictionaries, encyclopedias, monographs).

While most librarians agree (more or less) upon these procedures, the challenging task for them is to change these general guidelines into more useful structured data about the CD-ROMs. These data are used for prioritizing and selecting the most suitable CD-ROMs for the library collection.

Take Two at a Time

The pairwise comparison method utilizes human expert's preferences and judgments. These preferences are aligned to pairs of criteria or objectives. Evaluations are translated by a decision maker or an expert into a numerical (see Table 1 below) coding scale. We must stress the qualitative (not quantitative) importance of the scale. Numbers presented in the table are codes (i.e. symbols) for our comparative judgements. The method would work the same way for descriptive judgments (such as *very important* or *insignificant*), however, using codes for the intensity of importance in computer implementation is much more convenient.

Intensity of Importance	Definition	Judgement
1	Equal importance	Two criteria contributed equally to the objective
2	Weak importance of one over another	Experience and judgments slightly favour one criterion over another
3	Essential or strong importance	Experience and judgments strongly favour one criterion over another
4	Demonstrated importance	The criterion is strongly favoured and its dominance is demonstrated in practice
5	Absolute importance	The evidence favouring one criterion over another is of the highest possible order of affirmation
1.5, 3.7,... etc.	Intermediate values between the two adjacent judgments	When compromise is needed

Table 1. The equivalence between numerical codes and verbal statements

Input information is arranged in a pairwise comparison matrix **A** = a_{ij}, whose elements are all positive. They represent the intensities of a decision maker's preference between individual pairs of alternatives or criteria chosen from Table 1. The criteria A_1, A_2,..., A_n (n is the number of criteria to be compared). The entry a_{ij}, in i-th row and j-th column of the matrix **A**, denotes the relative importance of the criterion (objective) A_i compared with objective A_j, as expressed by a decision maker or by an expert. This matrix has all positive elements and has the following reciprocal property:

$$a_{ij} = \frac{1}{a_{ji}}$$

Let w_i denote the unknown weight of the criterion i. How can the vector $\mathbf{w}=[w_1,w_2,...,w_n]$ be estimated on the basis of the matrix **A**? One possible solution can be the following. If the decision maker's or the expert's assessment would be completely consistent, one would have

$$a_{ij} = \frac{w_i}{w_j}$$

for all pairs (i,j).

The last equation leads to the important meaning of cardinal consistency. The judgments A_i, A_j and A_k are said to be consistent if $a_{ij}*a_{jk} = a_{ik}$. In this case, the triad (A_i,A_j,A_k) is said to be consistent. The pairwise comparisons matrix **A** is consistent if all possible triads are consistent, i.e., the following equations holds:

$$a_{ij} = \frac{a_{ik}}{a_{jk}} \qquad i,j,k=1,2,....,n$$

It is not difficult to see that in this case:

$$\sum_{j=1}^{n} a_{ij} w_j = A \ w_j \quad for \ all \ i,j$$

which reads in matrix form:

$$Aw = nw$$

The last expression is an *eigenvector* expression, indicating that n is the largest *eigenvalue* of matrix **A**, and **w** is the corresponding eigenvector. This result holds true in the case of complete consistency. This is no longer true, however, in the case of inconsistencies, Saaty [Saaty, 1977] therefore, proposed an estimation of **w** by the eigenvector corresponding to the largest eigenvalue when there is inconsistency in matrix **A**. Inconsistencies often arise due to the complexity of the decision environment and the hesitation of perceptions and precision of expressing the judgement. The reality is, that an expert's or a decision maker's preferences are inconsistent. The problem of coping with inconsistencies is comparable to checking that the divisor in a planned division operation is not equal to 0. Simply, it does not make sense to divide anything by 0 and all proposed (heuristic) solutions to pairwise comparison models are based on an assumption that the given reciprocal matrix is consistent (see Saaty, 1977). A new definition of consistency [Koczkodaj, 1993] allows us to locate the most inconsistent judgments and reexamine them. New and more consistent judgments may be expressed in an interactive way, contributing to the overall reduction of the inconsistency.

It can be shown that the largest eigenvalue is never smaller than n. Moreover, small changes in matrix **A** cause small changes in the eigenvector but it is not shown that small changes in eigenvector cause small changes in matrix **A** [Saaty, 1980].

CD-ROM Selection Model

The Library Selection Committee usually needs to consider quite a large number of criteria, factors or alternatives during a CD-ROM selection process. Assigning the weight to each criterion is difficult. Having, for example, 100 points to be distributed among all criteria (that is assigning the importance of a criterion in percentage points), each librarian would probably face a difficult task. The obtained result must have a high degree of credibility. Are the librarians absolutely sure that the weights they have obtained are indeed the best and most reliable? The task becomes much more difficult when we have a number of experts coming from different fields. This is usually the case at a university library. Some of them are in favour of one group of criteria and others may prefer another group of criteria. How can we find a compromise to satisfy all experts? How can we solve the existing conflict of interests? These types of questions are addressed by the theories of Multiple Criteria Decision Analysis, Multiple Attribute Decision Making, and Concordance Analysis. The theoretical foundations were established at the beginning of the 1970's. The interested reader can find more information in the literature of [Nijkamp et al., 1990] [Ching-Lai and Kwangsun, 1981].

From another point of view, the famous Arrow's general possibility theorem indicates that there is no solution for the pure ranking method. For example, imagine a case with three criteria C1, C2, C3 and a team of 3*N (N is an integer number) stubborn human experts. In each triple, one expert ranks the criteria in the decreasing order of importance as [C1, C2, C3], one expert ranks criteria [C2, C3, C1], and one of them ranks as [C3, C1, C2]. (Please note the cyclic rotation of the criteria). It is easy to see that the above situation is the perfect tie and there is no possibility (or social function according to Arrow's terminology) to state which criterium is the most (or the least) important. It is quite important that the additional factor necessary for breaking the tie can be established by the pairwise comparison method. By asking each expert to express his/her relative preferences, that is ratios C1/C2, C2/C3, and C3/C1, we get something more than by asking to arrange C1, C2, and C3 in a decreasing (or increasing) order. In fact, an unsolved case for ratios exists when an expert insists on giving the same value for all ratios (for example 2; we may take any other figure without changing the generality). From C1/C2=2 and C2/C3=2 we have obtained (by multiplication of both sides) C1/C3=4 while it was originally evaluated to 0.5 (since C3/C1 was evaluated to two). It is worthwhile to notice that the original judgments were not only inconsistent (a frequent case of claiming that each and every factor is more important than the next factor!) There is no expertise to say which ratio is wrong: C1/C2, C2/C3, or C3/C1.

In the presented model we need to be prepared for a certain level of inconsistency in our judgments [French, 1986, Sen, 1977]. Zero tolerance of inconsistency is tolerated or even desirable. It is unrealistic to assume that our judgments are fully consistent. It may harm the selection process which should be aiming towards decreasing inconsistency of our judgments (by a refinement process) without, however, a temptation of chasing the inconsistency for the complete zero. The fuller analysis of the new inconsistency is presented in [Koczkodaj, 1993].

After a preliminary discussion we shortlisted the following criteria listed in alphabetical order:
- additional software requirements,
- collections development tools,
- ease of use,
- full-text,
- increase of usage,
- indexes,
- monograph,
- networkable,
- reference tools,
- relevance to academic program,
- size of user population,
- source,
- substitution by online source,
- substitution by print source,
- update frequency.

It is worthwhile to note the total flexibility and diversity of the criteria. In fact, it is advisable to list as much of the various criteria as possible (including very exotic cases) and gradually eliminate them by group discussion (Delphi method, for example, could be used in case of close ties). The above list is a refined product but it can be altered to fulfil each library's individual need and expectation (e.g. more stress to research or teaching will quite likely cause changes in the length and contents of the above list).

The proposed model is based on the modified theory of *pairwise comparisons*. The number of criteria is too large for looking at all possible combinations of pairs (in our case 105; 15*(15-1)/2 is the number of all combination of pairs made of 15 different items). In practice it means that we need to group the criteria and apply the pairwise approach to groups. Instead of comparing all criteria at once, the experts will compare criteria in groups in one level of the hierarchy at a time.

Cooperating with the experts from the Library of Laurentian University we agreed to consider four groups of criteria. The basis for grouping was from their opinion of the library's need. From the pairwise comparison method point of view it would be better to have three groups of equal length.

The first group ***Format*** indicates a type of publication. The following entries in this group are: *indexes* (understood as sources to journal literature in a discipline; some indexes include book chapters and book reviews), *reference tools* (regarded as a factual information sources; they may be single or multiple tools), *collections development sources* (tools to be used for searching available materials *monographs* (as a single issue publication, for example, Shakespeare's Plays).

The second group ***Contents*** exhibits an intellectual content. The following entries are included in this group: *source* (understood as persons or organizations responsible for the intellectual content), *full-text* (determines how well the CD-ROM content substitutes the full text), *relevance to academic program* (assessment of a support for courses, library functions and strategic plans), *update frequency* (evaluation how often new or revised information is added to the database).

The third group ***Usage*** assembles entries related by whom and how the CD-ROM is utilized and usage impact. The entries in this group are: *ease of use* (evaluation of the user interface, e.g. search platform menu or command driven), *size of user population* (estimation of the potential number of students or faculty or staff users), *increase of usage* (will the CD change the use of our existing "print" journal and book collections?).

The fourth group ***Technical factors*** describes various equipment/software characteristics. It has the following entries: *networkable* (can the CD be accessed by more than one person at a given time?), *additional software requirements* (does the CD need, for example DOS extensions or additional installation and setup programs), *substitution by online source* (are there public domain or commercial databases as a source for similar information?), *substitution by print source* (evaluation of a comparable print version).

Figure 1. The hierarchy tree of all entries

After defining the model, the entries in groups are rearranged by a visual sort in descending order. The user changes position of the entry by dragging it with a mouse. Next, the set of questions is generated by the software system for each group. All possible combinations of pairs were exhausted. For a group with three entries there are only three possible combinations (that is 3*(3-1)/2) while for a group with four entries we have six different combinations (that is 4*4-1)/2). Each question asks the experts to compare two entries in the same group for their relative importance. The software system calculates the weights of all criteria (factors). It has been mathematically proven [Saaty, 1980] that these weights are the most accurate estimates for the given answers (judgments). An example of a question related to entries is "How many times is *the ease of use* more important than the *size of the user population*?"The pairwise comparison process starts by comparing the groups between each other (there are only six combinations for four groups). For example, "Estimate relative importance of the group *Format* with the group *Technical factors*?

We have used in our model the scale presented in Table 1. The following verbal statements approximate the experts' opinions (the numerical codes of each statement are placed in parenthesis). In case of any doubts the "equal importance" answer is recommended. Code (1) has an additional interpretation: "Lack of opinion, no preference as far as these two factors are concerned).

- equal importance of both criteria (1),
- weak importance of one criterion over another (2),
- essential importance of one criterion over another (3),
- demonstrated importance of one criterion over another (4),
- absolute importance of one criterion over another (5).

Intermediate answers express judgments which belong in between two categories and should be considered as a case of certain uncertainty or hesitation toward either opinion. They allow for further flexibility.

Tables 2, 3, 4, 5, and 6 below, show answers collected from the library experts at the Laurentian University Library. All experts' judgments are placed in pairwise comparison matrices. Asterisk (*) denotes cases which are trivial (importance of a pair with the same entry, e.g. *format* against *format*, which is always 1), or could be obtained from the existing data (no need to have opinion on *contents* against *format* since we have opinion on *format* against *contents*.

Is more important than --------->	Format	Contents	Usage	Technical
Format	*	1.0	3.0	4.0
Contents	*	*	2.0	4.0
Usage	*	*	*	3.0
Technical	*	*	*	*

Table 2. CD-ROM selection group

Is more important than --------->	Indexes	Collection dev. tools	Ref. tools	Monogr.
Indexes	*	2.5	3.5	5.0
Coll. dev. tools	*	*	1.5	4.0
Ref. tools	*	*	*	3.0
Monogr.	*	*	*	*

Table 3. Format group

Is more important than --------->	Relevance ac. coll.	Source	Update frequency	Full text
Relevance to ac. coll.	*	2.0	1.0	4.0
Source	*	*	1.0	3.5
Update frequency	*	*	*	3.0
Full text	*	*	*	*

Table 4. Contents group

Is more important than ---------->	Ease of use	Size of user pop.	Increase in usage
Ease of use	*	3.0	2.0
Size of user pop.	*	*	1.0
Increase in usage	*	*	*

Table 5. Usage group

Is more important than --------->	Subst. by print	Subst. by on-line	Network.	Addit. soft. requir.

Subst. by print	*	2.0	3.0	4.0
Subst. by on-line	*	*	2.0	3.0
Networkable	*	*	*	3.0
Addit soft. requir.	*	*	*	*

Table 6. Technical group

Consistency Analysis

The definition of consistency of a pairwise comparison matrix A, based on eigenvalues, was introduced by Saaty [Saaty, 1977]. The consistency factor is given by the following formula:

$$cf=\frac{\lambda_A - order(A)}{(order(A)-1)\ \lambda_{random}}$$

However, the above formula leads to some theoretical problems [Shen, 1990] of which the biggest deficiency is the unknown location of the inconsistency. In the presented model the new definition of consistency [Koczkodaj, 1993] is used. It is based on one triad (A_i,A_j,A_k) of the comparisons matrix **A**. In this case, the pairwise comparisons matrix reduces to the following 3 by 3 basic reciprocal matrix **A3**

$$A3(A_i,\ A_j,\ A_k)=\begin{bmatrix} 1 & a & b \\ 1/a & 1 & c \\ 1/b & 1/c & 1 \end{bmatrix}$$

where a expresses a referee's relative preference of criterion A_i, over A_j, b expresses preference of criterion A_i, over A_k, and c is a relative preference of stimulus A_j, over stimulus A_k. Matrix **A3**(A_i,A_j,A_k) is consistent if, and only if, b=a*c.

The new definition of the consistency of a basic reciprocal matrix **A3**(A_i,A_j,A_k) is based on the following intuition: it is a measure of deviation from the nearest basic consistent reciprocal matrix. The interpretation of the consistency measure becomes more apparent when we reduce a basic reciprocal matrix to a vector of three coordinates [a,b,c]. We know that b=a*c holds for each consistent reciprocal matrix, therefore, we can always produce three consistent reciprocal matrices (therefore three vectors) by computing one coordinate from the combination of the remaining two coordinates. These three vectors are: [b/c,b,c], [a,a*c,c], and [a,b,b/a]. The inconsistency measure will be defined as the relative distance to the nearest consistent reciprocal matrix represented by one of these three vectors for a given metric. In the case of Euclidean (or Chebysheff) metrics we have:

$$CM(A_i,\ A_j,\ A_k)\ =\ \min\left(\ \frac{|a-\frac{b}{c}|}{a},\ \frac{|b-a*c|}{b},\ \frac{|c-\frac{b}{a}|}{c}\ \right)$$

for all a,b,c $\geq$ 1.

The above definition can be extended to a pairwise comparison matrix of any order by the following formula:

$$CM(\mathbf{A})\ =\ \max(\ CM(A_i,\ A_j,\ A_k)\ :\ 1<i<j<k<n\ )$$

This definition gives the opportunity to build a tool for reducing the inconsistency of the expert's judgments. It should be seen as a technique for data validation in knowledge acquisition. The measure of the validity of knowledge is the consistency measure of a comparison matrix. To "improve" the quality of the knowledge, an expert, with the help of computer software, might calculate the consistency measure. The computer system is pointing to the triad with the highest inconsistency, allowing experts to reconsider their judgments. The important point is, that the system does not force the experts to change their judgment. Instead, the computer program shows the experts the most inconsistent elements of their judgements.

Figure 2. The triad with the most inconsistent judgments

Figure 2 shows that the following three judgments are the most inconsistent: ***format*** is three times more important than ***usage***, ***format*** is four times more important than ***technical factors*** and ***usage*** is three times more important than ***technical factors***. Moreover, inconsistency measure of this pairwise comparisons matrix is equal to 56%.

After changing the judgment, ***format*** is three times more important than ***usage***, to the judgment ***format*** is one and a half times more important than ***usage*** we decreased the inconsistency measure of this pairwise comparisons matrix to 33% and we received the following inconsistencies:

Figure 3. The triad with the most inconsistent judgments after consistency analysis

After consistency analysis the pairwise comparison matrix of all groups of factors has been transformed to:

Is more important than --------->	Format	Contents	Usage	Technical
Format	*	1.0	1.5	4.0
Contents	*	*	2.0	4.0
Usage	*	*	*	3.0
Technical	*	*	*	*

Table 7. CD-ROM selection group after consistency analysis

We have received the following weights of all criteria:

Figure 4. The sorted list of weights of all criteria

We can see that the most decisive entries are:

- indexes (17.11%),
- relevance to academic program (13.32%),
- ease of use (9.94%),
- source (9.79%),
- update frequency (8.51%),
- collections development tools (8.28),
- increase of usage (6.89%),
- reference tools (6.21%).

They constitute more than 80 percent of the total score. Using the selected factors saved the committee's time since it is essentially easier to evaluate 8 rather than 15 factors for each title.

A careful reader will notice that factors related to budget have not been included in our model. Is it a simple overlooking? Or, perhaps, lack of authors' believe in monetary values. We are fully aware of the severe budgetary constraints at most college and university libraries and without any hesitation we may say that the entire project was directed to this goal. It is not, however, reasonable, to try to propose any equation between scientific merits of a CD-ROM item and its cost. Having them separated (costs of CD-ROM item is stored as a separate variable in our model) makes our model more flexible and powerful.

The final selection of CD-ROM titles nearly always sharply comes to a point which can be illustrated as a child's dilemma of what to wash, the hands or the legs. Is it better to buy one title for $10,000 or ten titles at $1,000? In real-life, it all depends upon the library merit is of each case. An automatic selection by any automatic algorithm (e.g. saturated greedy algorithm outlined below) may distort the allocation process.

The used saturated greedy algorithm has its roots in the Radon-Nikodym theorem [Halmos, 1962]. First we sort all evaluated candidate CD-ROM items in a decreasing order according the library merits (starting with the CD-ROM with the highest score) as described earlier in the proposed model. Having a given budget limit (e.g. $200,000) we pick up the first item on the list and allocate the budget provided we have enough funds for it, otherwise we try it with the next item on the list. We repeat our procedure unless we exhaust the entire budget (which usually comes faster than we wish). The solution produced by the greedy algorithm tends to be nearly optimal under wide circumstances. In general, when the number of CD-ROM titles is high while their prices are relatively low to the budget, then the selection produced by the greedy algorithm is still a nearly optimal (or even an optimal) solution. Even more generally, the selection is a nearly optimal solution when the remaining CD-ROM items have prices which are relatively low when compared to the budget [see, Holsztynski and Koczkodaj, 1994].

In practice, a human intervention is required at a certain point unless we are prepared to specify "library merits" for each candidate title (this task may not be possible to complete given complexity of the "library merit" and a number of existing titles on the market). A fine tune up of the final list produced by the above described (and easy to implement) saturated greedy algorithm is nearly always required. Usually, it is done by a library committee or qualified librarians. Our model fully supports such approaches by allowing changing priorities for different characteristics (e.g. reference tools versus indexes). Fights for squeezing "my title" in the collection are unavoidable no matter which method is used. We hope, however, that our model will contribute to a more humane solution when a list with library merits is presented to a budget allocation constituency.

Conclusions

Using the consistency-driven model allows librarians to cut time on needless discussions on what is the best CD-ROM collection for the library. Such discussions are inconclusive and usually they result with approving the purchase of the existing collection (for the convenience of the financial administration rather than for the library readership) and adding some absolutely necessary new titles.

The presented model contributes essentially to the quality of a selected collection by using the new definition of consistency which allows us to locate the inconsistency. The selection committee is given the feedback and opportunity of reconsideration of judgments. The model is flexible and adoptable to new environments. The authors invite comments and inquiries by electronic mail to waldemar@ramsey.cs.laurentian.ca or icci@nickel.laurentian.ca. Free copies of the software running under DOS on PC compatible computers will be sent to those willing to use or test our system. Further improvements to the model are welcomed and expected from future users. Wiser selection of highly expensive CD-ROM products is a necessity in this harsh economic time. It is in our best interest to have more than less for the same and usually shrinking amount of funds allocated to CD-ROM collections.

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