



Comment

Approximate reasoning by pairwise comparisons
“Topodynamics of metastable brains” by Arturo Tozzi, et al.

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The innovative approach in [1], “Topodynamics of Metastable Brains” by Arturo Tozzi, James Peters, Andrew Fingelkurts, Alexander Fingelkurts, and Pedro Marijuan has a high potential of becoming a paradigm shift in the brain research. It seems that this study has successfully explored the possibility of applying a celebrated Borsuk–Ulam theorem to the operational architectonics of the fundamental brain-mind processes.

It has been already in use in practically all branches of dynamics in classical mechanics, quantum physics, fluid and gas dynamics. Among the most recognized names contributing to this approach are Sophus Lie and Henri Poincaré. In our opinion, [2] provides a comprehensive introduction. An exposition on holonomy is given in [3] and in the context of pairwise comparisons in [4]. Fig. 1 shows the parallel transport which is an illustration of a holonomy (one of the invariants). The first step in the discussed method is to identify quantities related to the dynamics in order to describe the internal structure of the analyzed system. These quantities can have several names which carry very explicitly the corresponding concepts: symmetries, invariants, characteristic classes, curvatures (just to name a few of them). In fact, there is a plethora of such “invariants” expressing some obstructions, or the ability to get some kind of stability or structure. Non-linear systems are not stable enough to be sufficiently analyzed by classical numerical methods. For them, invariants are of particular use. To the best of our knowledge, the most refined approach (in this spirit) appears in the mathematical quantum physics (see [3] for more on this subject).

By “topodynamics”, the authors signify hidden structures of metastable brains via symmetries. The principal topological tool is the Borsuk–Ulam theorem (BUT). It is strongly related to the theory of invariants and symmetries. More precisely, it is a part of fixed-point theory. BUT is also related to symmetries on a hypersphere which are special

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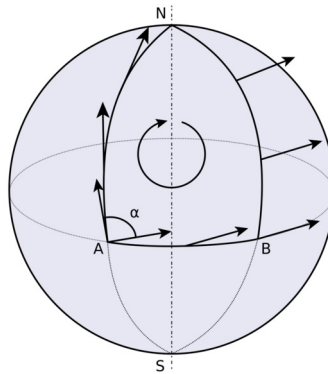


Fig. 1. Parallel transport.



Fig. 2. Stone age decision.

cases of compact manifolds. Hidden structures elevate explicit symmetries, which can be verified via experience. These hidden structures can be intrinsic, or extrinsic. From the historical viewpoint, the most famous hidden structure of equations and dynamical systems has been developed by Evariste Galois via so-called Galois theory. Galois theory shows how complex, promising and powerful this approach can be. A. Tozzi et al. [1] demonstrate how the Borsuk–Ulam theorem and its generalizations can provide an innovative approach to analyzing metastable brain-mind dynamics in terms of projections from real to abstract phase spaces. Their findings are of considerable importance for designing mathematical and computational models of the fundamental brain-mind processes. The concept of antipodal points, in mathematics, is generalized to spheres of any dimension. Two points on such a sphere are antipodal if they are placed on the opposite sides of the sphere through its center. Evidently, antipodal points are diametrically opposite as each line connecting them goes through the center of the sphere. Needless to say that two points create a pair. Hence, the method of pairwise comparisons, especially the inconsistency in pairwise comparisons (recently published in [5], mathematically analyzed in [6,4], and previously linked to perceptual and motor skills in [7]) may be explored as a potential enhancement. Approximate reasoning methods are applicable to processing inconsistent data. Inconsistency is related to imprecision and uncertainty which often occur in subjective assessments. However, subjective assessments are unfairly neglected as unreliable or biased yet, without them, it is hard to imagine the progress of science since even awarding scientific degrees is impossible without subjective expert opinions. There is no “yard stick” for measuring such criteria as “expert level” yet no modern society could function without recognizing some of its members as experts (e.g., expert witness). In the absence of well-established units of measurement, pairwise comparisons (PCs) are of great help. In such a case, the smaller entity often becomes the implicit “unit” and we use the linguistic expression “is x times larger (or more important) than the unit” to express the size of the larger entity. The first application of pairwise comparisons in the 13th century has been recently attributed to Llull (see [5]). Deciding which of two objects (e.g., stones) may fit for purpose is a very natural question and undoubtedly took place, in an intuitive way, during the Stone Age (Fig. 2).

As noted, pairwise comparisons (PCs) could be linked with the “topodynamics” approach as evidenced in a very recent ([4]) geometric interpretation of PCs in terms of the differential-geometric notion of holonomy. [4] opens the road

to recognition of classical topological–geometric invariants in inconsistency where the same invariants give characteristic classes of simple geometric structures such as closed compact manifolds which are a part of the Borsuk–Ulam theory, highlighted in the study of Tozzi et al.

Finally, our brain has a pair of hemispheres. It gets input from a pair of eyes and ears. The brain controls the movement of two hands and legs. Even our reproductive systems needs two humans (hence a pair) to create a new life. Is it all coincidence or the pairs rule the life?

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