Quintessence and Majorana Neutrinos: Proposal for Unification of the Dark Sector

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References:

1. G.Y. Chitov, T. August, N. Aravind, T. Kahniashvili, PRD (2011)
2. G.Y.C. , ArXiv:1112.4798

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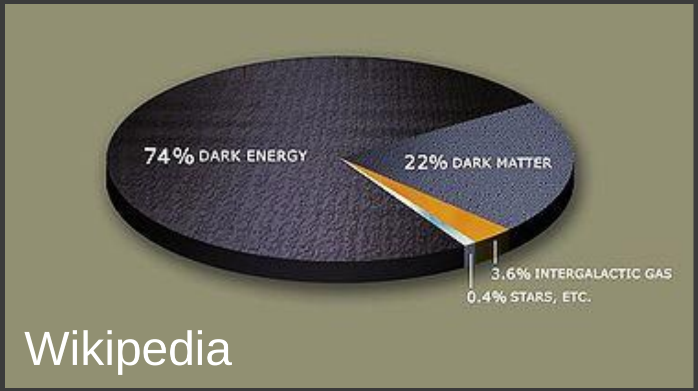
Outline:

- Motivation and Introduction
- Model and Formalism
- Three Phases of the Universe (Stable, Metastable, Unstable). Key Results, Parameters of the Model
- Dynamics of the Model and Observable Universe
- Extension of the Standard Model: Majorana Neutrinos, Seesaw
- Conclusions

Composition of the Universe: Bookkeeping

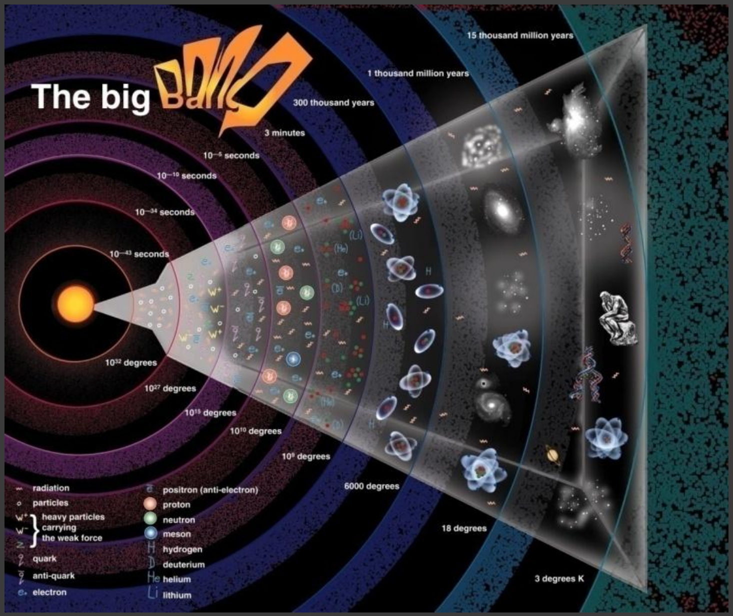


Wikipedia

$$\rho_{DE} \approx 30 \cdot (10^{-3} eV)^4$$

$$\rho_{DE} \sim \rho_{DM}$$

$$m_\nu \sim 10^{-1} - 10^{-3} eV$$



DE/DM-dominated era

References:

S. Weinberg, *Cosmology* (Oxford University Press, New York, 2008).
S. Dodelson, *Modern Cosmology* (Academic Press, San Diego, 2003).
P.J.E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003).
S.M. Carroll, eConf C0307282, TTH09 (2003); AIP Conf. Proc. **743**, 16 (2004).
E.J. Copeland, M. Sami, and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006).

Dark Energy and Cosmological Constant

Einstein (1917) $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} .$$

$$\rho = \frac{\Lambda}{4\pi G} ,$$

$$\tilde{\rho} = \rho + \frac{\Lambda}{8\pi G} , \quad \tilde{p} = p - \frac{\Lambda}{8\pi G}$$

$$\Lambda - term : \rho = -p$$ ⟷ $$a \propto e^{Ht}$$

↓

Dark Energy, Anti-Gravity ("Gravitational Repulsion")

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p)$$

DE and Cosmological Constant:
(1) Fine Tuning Problem

Observations:
(1998)

$$\Lambda \approx H_0^2 = (2.13h \times 10^{-42}\,\mathrm{GeV})^2$$

$$\rho_\Lambda = \frac{\Lambda m_{\rm pl}^2}{8\pi} \approx 10^{-47}\,\mathrm{GeV}^4$$

Riess A G *et al* 1998 *Astron. J.* **116** 1009–38
Perlmutter S *et al* 1999 *Astrophys. J.* **517** 565–86

$$10^{121} \text{ orders of magnitude}$$

QFT & other theories:
(vacuum energy)

$$\rho_{\rm vac} = \frac{1}{2}\int_0^\infty \frac{d^3k}{(2\pi)^3}\sqrt{k^2+m^2} \approx \frac{k_{\rm max}^4}{16\pi^2}$$

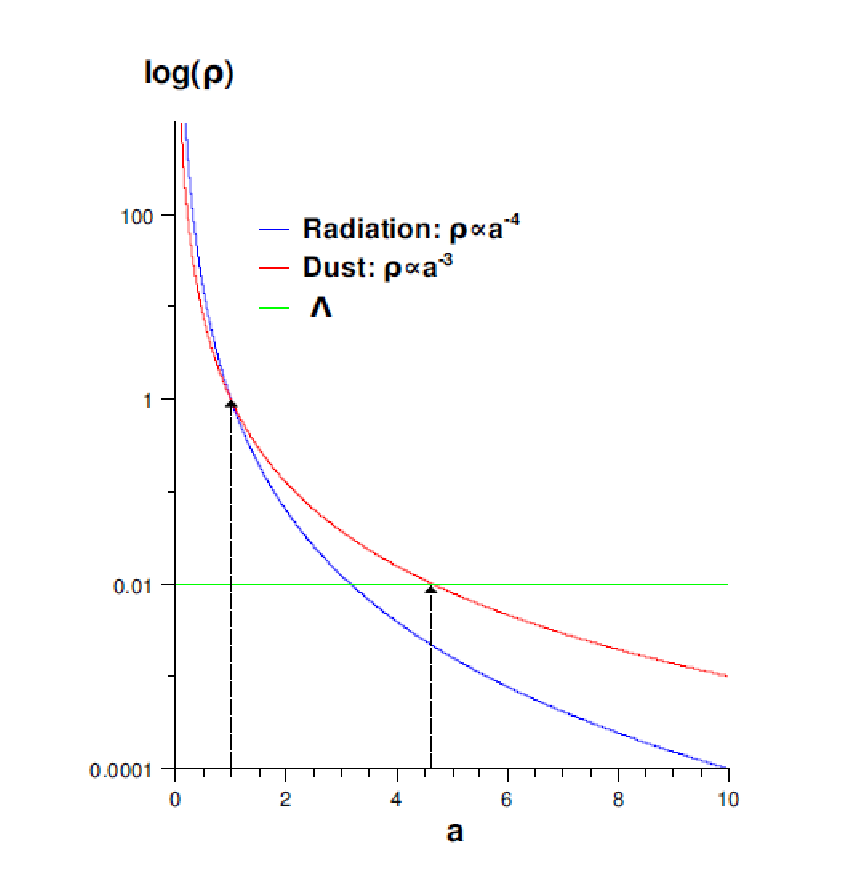
$$\rho_{\rm vac} \approx 10^{74}\,\mathrm{GeV}^4$$

$$m_{\rm pl} = 1.22 \times 10^{19}\,\mathrm{GeV}$$

This is a standard argument, however misleading (!!)

DE and DM:
(2) Coincidence Problem

$$\rho_{DE} \sim \rho_{DM}$$



- At current time, densities of DE, DM and cosmological neutrinos are roughly comparable.
  "Narrow Coincidence": DE density and neutrino mass are controlled by the same mass scale M

$$\rho_{DE} \sim M^4 \qquad m_\nu \sim M$$

- The origin of neutrino mass is not explained by the Standard Model

DE as Quintessence: $\Lambda \Rightarrow U(\varphi)$

1) GR is correct theory
2) True vacuum of the Universe (a la Volovik (?))

$$\rho_{DE} = 0 \ (\Lambda = 0)$$

3) DE=Quintesseence (Wetterich, 1988; Ratra & Peebles, 1988)

$$DE \Rightarrow \mathrm{U}(\varphi)$$

4) Ratra-Peebles DE Potential

$$U(\varphi) = M^{\alpha+4} / \varphi^{\alpha}$$

Scalar Field – Quintessence
(Fifth Element/Force)

1) Gravity
2) E&M
3) Strong
4) Weak
5) ???

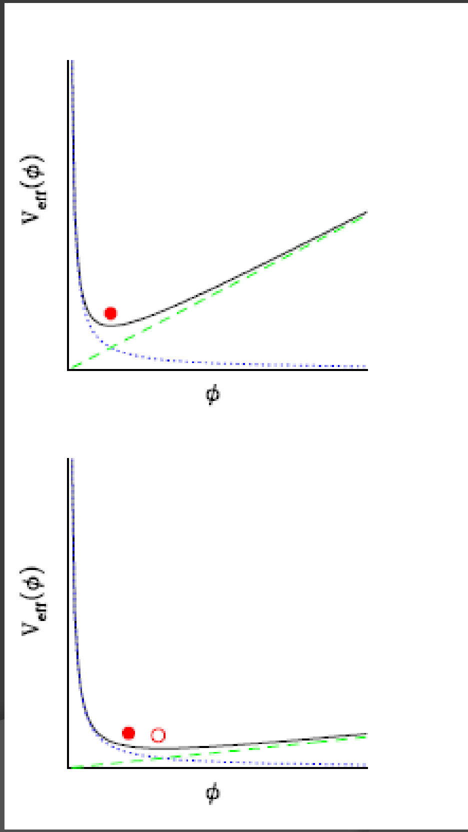
Varying Mass Particles (VAMPS)

Ingredients:

1) Scalar field (Quintessence) → DE
2) Massless Particles
3) Yukawa coupling $\propto \varphi\bar{\psi}\psi$

$\langle\varphi\rangle \Rightarrow m$ VAMPs

Anderson & Carroll, 1997
Hoffmann, 2003



Mass-Varying Neutrino (**MaVaN**) Scenario
Fardon, Nelson & Weiner (2004)

Trouble (!!!): Instability

Solution (???)

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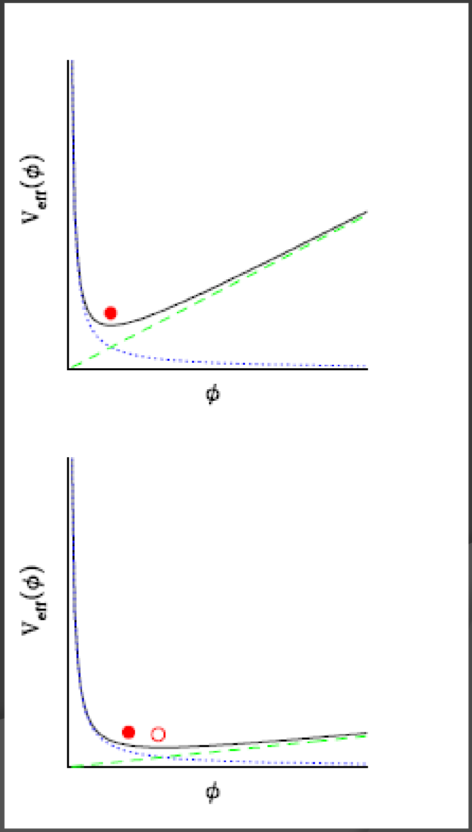

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Mass Varying Neutrino Scenario (MaVaN):

(1) We study the case when the quintessence potential U does not have a non-trivial minimum

→ the generation of the fermion mass is due to breaking of the chiral symmetry in the Dirac sector of the Lagrangian.

(2) We assume the cosmological evolution governed by the scalar factor a(t) to be slow enough:

→ The system is at equilibrium at a given temperature T(a)
→ The methods of the thermal quantum field theory can be applied.

(3) We study possibly the simplest "*minimal model*":

→ fermions are described by the Dirac spinor field
→ zero chemical potential

Model and Formalism:

The Euclidian action of the model in the FLRW metric:

$$\mathcal{S} = S_B^E + S_D^E\big|_{m=0} + g\int_0^\beta d\tau \int a^3 d^3x\ \varphi\bar{\psi}\psi$$

$$S_B^E = \int_0^\beta d\tau \int a(t)^3 d^3x\ \left[\frac{1}{2}(\partial_\tau\varphi)^2 + \frac{1}{2a^2}(\nabla\varphi)^2 + U(\varphi)\right]$$

$$S_D^E = \int_0^\beta d\tau \int a(t)^3 d^3x\ \bar{\psi}(\mathbf{x},\tau)\left(\gamma^o\frac{\partial}{\partial\tau} - \frac{\imath}{a}\gamma\cdot\boldsymbol{\nabla} + m - \mu\gamma^o\right)\psi(\mathbf{x},\tau).$$

The partition function of the coupled model

$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\bar{\psi}\mathcal{D}\psi\ \mathrm{e}^{-\mathcal{S}}$$

The Ratra-Peebles quintessence potential

$$U(\phi) = \frac{M^{\alpha+4}}{\varphi^\alpha}$$

Saddle-Point Approximation ←→ Min of the (Grand) Thermodynamic Potential

$$\frac{\partial\Omega(\phi)}{\partial\phi}\bigg|_{\phi=\phi_c} = 0$$

Mass Equation and Critical Temperatures:

$$U(\phi) = \frac{M^{\alpha+4}}{\varphi^{\alpha}}$$

It is convenient to introduce the dimensionless parameters

$$\Delta \equiv \frac{M}{T} \;, \quad \kappa \equiv \frac{g\varphi}{T} \;, \; \Omega_R \equiv \frac{\Omega}{M^4} \;.$$

Mass equation:

$$\frac{\alpha\pi^2}{2\mathfrak{s}} g^{\alpha} \Delta^{\alpha+4} = \mathcal{I}_{\alpha}(\kappa)$$

$$\mathcal{I}_{\alpha}(\kappa) \equiv \kappa^{\alpha+2} \int_{\kappa}^{\infty} \frac{\sqrt{z^2 - \kappa^2}}{e^z + 1} dz$$

$$\frac{\alpha\pi^2}{2\mathfrak{s}} g^\alpha \Delta^{\alpha+4} = \mathcal{I}_\alpha(\kappa)$$

$$\Delta \equiv \frac{M}{T}\ , \quad \kappa \equiv \frac{g\varphi}{T}\ , \quad \Omega_R \equiv \frac{\Omega}{M^4}$$

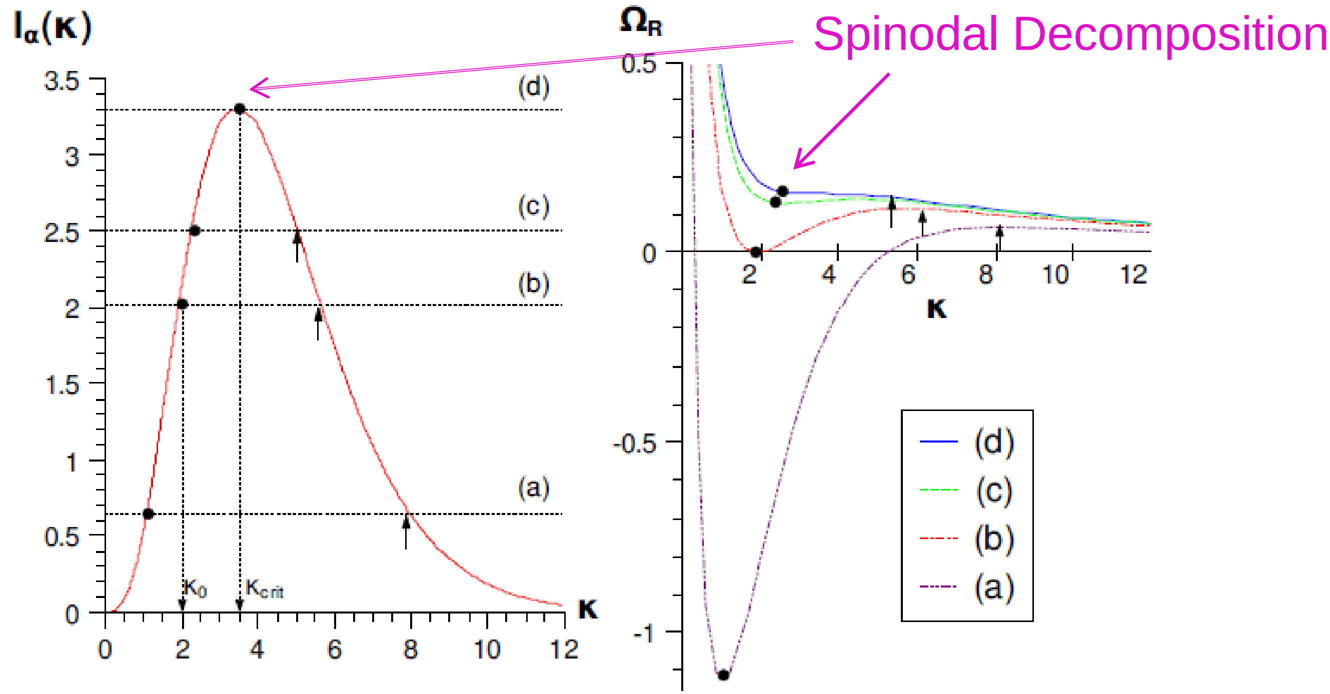

FIG. 1: (Color online) Left: Graphical solutions of the mass equation (50) for different values of $\Delta \equiv M/T$ ($g = 1$, $\alpha = 1$). Right: Dimensionless density of the thermodynamic potential

There are 3 phases:

(1) Stable (T>>M)

(2) Metastable (T~M)

(3) Unstable (T<M)

via First-Order Phase Transition

Velocity of Sound & Stability:

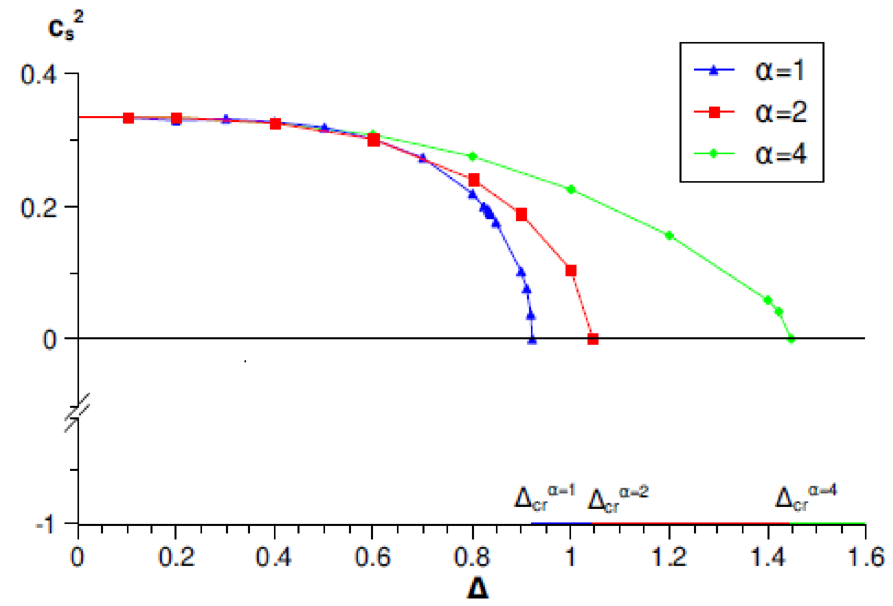

FIG. 4: (Color online) The square of the sound velocity for several values of α. At $\Delta > \Delta_{\rm crit}(\alpha)$, i.e., $T < T_{\rm crit}(\alpha)$ the equilibrium value $c_s^2 = -1$ exactly.

Masses vs. Temperature:

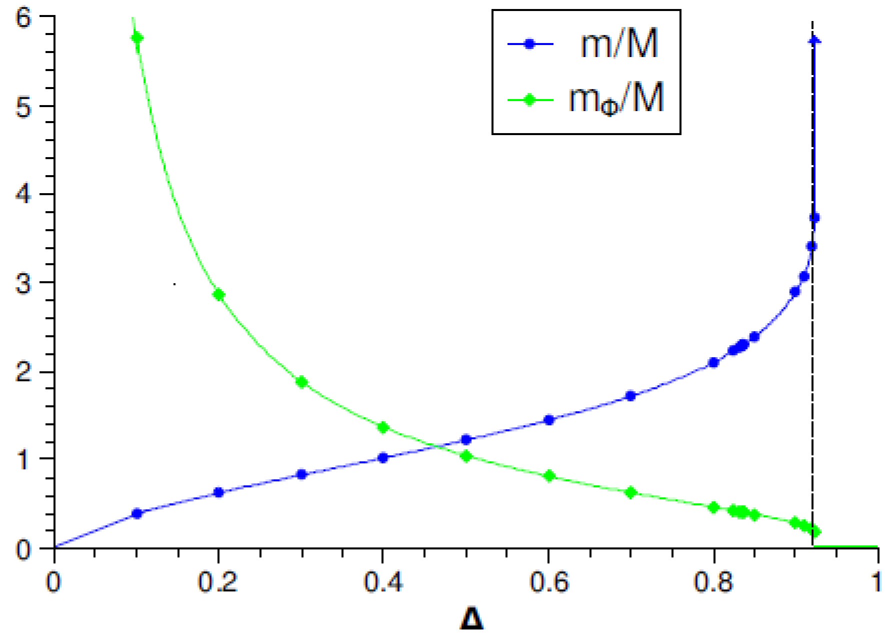

FIG. 2: (Color online) Masses of the fermionic and scalar fields (m and m_ϕ resp.) as functions of $\Delta \equiv M/T$, $\alpha = 1$. At $\Delta > \Delta_{\rm crit}$ ($T < T_{\rm crit}$) the stable phase corresponds to $m = \infty$ and $m_\phi = 0$

Dynamics of the Model and Observable Universe

- *Currently we are below the critical temperature (!!!)*

The equation of motion:

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{\partial \Omega}{\partial \varphi} = 0$$

Matter-dominated & Slow-rolling regimes:

$$\bar{\varphi} = \varphi_{\rm crit} \cdot \left(\frac{1+z_{\rm crit}}{1+z}\right)^{\frac{3}{\alpha+1}},$$

$$\rho_{\bar{\varphi}} = \rho_{\varphi,\rm crit} \cdot \left(\frac{1+z}{1+z_{\rm crit}}\right)^{\frac{3\alpha}{\alpha+1}}$$

$$\rho_{\rm now}(\varphi) = \frac{3}{4} \cdot \frac{3H_\circ^2}{8\pi G} \approx 31 \cdot (10^{-3}\ eV)^4$$

Single scale M (!!)

$$U(\varphi) = \frac{M^{\alpha+4}}{\varphi^\alpha}$$

$$U(\varphi) = M^4\left(1 + \alpha \log \frac{M}{\varphi}\right)$$

α	M (eV)	$m_{\rm now}$ (eV)	$z_{\rm crit}$	z^*
2	$9.75 \cdot 10^{-2}$	167	392	4.9
1	$1.69 \cdot 10^{-2}$	44.6	76.6	2.3
1/2	$6.33 \cdot 10^{-3}$	17.0	27.7	1.5
10^{-1}	$2.81 \cdot 10^{-3}$	2.82	8.73	0.93
10^{-2}	$2.39 \cdot 10^{-3}$	0.27	3.67	0.83
10^{-3}	$2.36 \cdot 10^{-3}$	0.027	1.60	0.82

$$\rho_{\rm tot} = \rho_{\gamma,\rm now}/a^4 + \rho_{M,\rm now}/a^3 + \rho_{\varphi\nu}(\Delta) = \rho_{\rm cr} = \frac{3H^2}{8\pi G}$$

$$\Omega_\# \equiv \rho_\#/\rho_{\rm tot}$$

$$\Delta \equiv \frac{M}{T} = \frac{Ma}{T_{\rm now}} = \frac{M}{T_{\rm now}(1+z)}$$

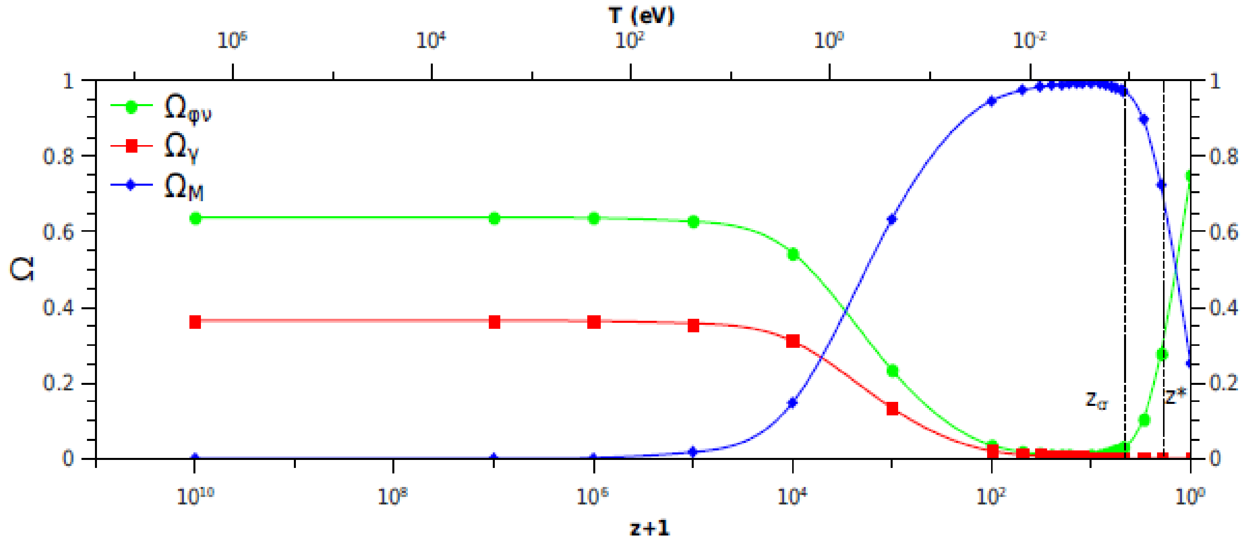

FIG. 5: (Color online) Relative energy densities plotted up to the current redshift (temperature, upper axis): $\Omega_{\varphi\nu}$ – coupled DE and neutrino contribution; Ω_γ – radiation; Ω_M – combined baryonic and dark matters. Parameter $M = 2.39 \cdot 10^{-3}$ eV ($\alpha = 0.01$), chosen to fit the current densities, determines the critical point of the phase transition $z_{\rm cr} \approx 3.67$. The crossover redshift $z^* \approx 0.83$ corresponds to the point where the Universe starts its accelerating expansion.

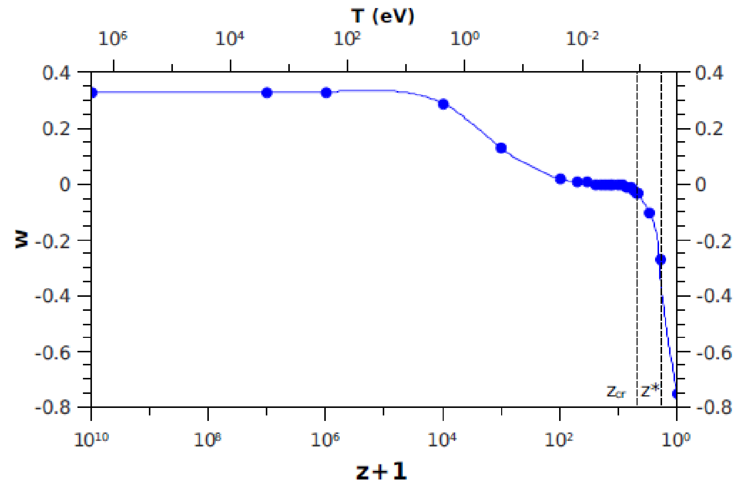

FIG. 6: (Color online) Equation of state parameter $w_{\rm tot} = P_{\rm tot}/\rho_{\rm tot}$ for $M = 2.39 \cdot 10^{-3}$ eV ($\alpha = 0.01$) plotted up to the current redshift (temperature, upper axis).

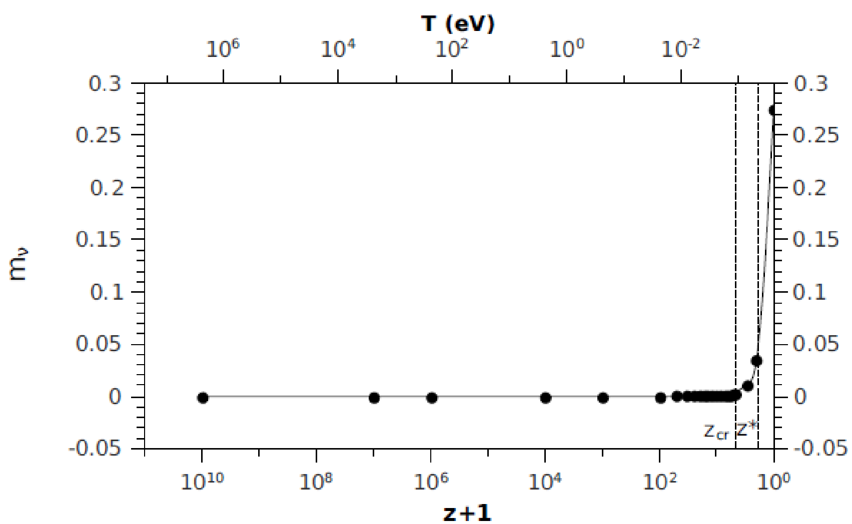

FIG. 7: Neutrino mass m for $M = 2.39 \cdot 10^{-3}$ eV ($\alpha = 0.01$) plotted up to the current redshift (temperature, upper axis).

Extension of the toy model – Towards Unification: Seesaw, Active /Sterile Majorana Neutrinos,DE

1. Toy Model (previous analysis): **WHERE IS DM ???**

2. Extension of the Model (a la nuMSM, Shaposhnikov et al, 2005-)

(i) Standard Dirac term in the Lagrangian:

$$\mathcal{L}_{YD} = m_D \bar{\psi}_R \psi_L + h.c.$$

where the Dirac mass is due to Yukawa coupling with Higgs scalar:

$$m_D = g_D \langle H \rangle \qquad (\langle H \rangle = 174 \text{ GeV})$$

$$\text{(light leptons) } 1 \text{ MeV} \leq m_D \leq T_{EW} \sim 10^2 \text{ eV}$$

(ii) Plus Right-Handed Majorana Neutrino term:

$$\mathcal{L}_{YM} = \frac{1}{2} m_R \bar{\psi}_R^c \psi_R + h.c.$$

where the Majorana neutrino mass is due to DE (quintessence)

$$m_R = g \langle \varphi \rangle$$

Results:

1. Two Majorana Fermions with eigen-masses:

(a – active, light)
(s – sterile, heavy)

$$m_{s,a} = \frac{1}{2}\left(\sqrt{m_R^2 + 4m_D^2} \pm m_R\right)$$

2. Minimization (mass) equation:

$$U'(\varphi) + \frac{1}{2}\frac{g}{\sqrt{m_R^2 + 4m_D^2}}\Big(m_s\rho_{ch}(m_s) - m_a\rho_{ch}(m_a)\Big) = 0$$

3. At high temperatures $T>T_{EW}$ (when $m_D=0$).
Similar to the toy model of a single Dirac fermion

active neutrino $m_a = 0$

sterile neutrino is light and relativistic $m_s = m_R \approx \mathcal{O}(1) \cdot M(M/T)^{2/(\alpha+2)}$

4. The is a first order phase transition (spinodal decomposition) at

$$T_c \approx T_{cD}\left(1+\zeta_\alpha\left(\frac{M}{m_D}\right)^2\right) \qquad T_{cD} = \frac{2}{5}m_D$$

$$m_R^c \approx \vartheta_\alpha M \qquad m_\phi^c \sim M \qquad \zeta_\alpha \sim \vartheta_\alpha \sim \mathcal{O}(1)$$

T / M, m_D	$T \geq T_{EW} \sim 10^2$ GeV $m_R = m_s \sim M\left(\frac{M}{T}\right)^{2/(\alpha+2)}$ $m_a = m_D = 0$	$T = T_c \approx 0.4 m_D$ $m_R^c \approx 6.42M$ $m_{s,a}^c \approx m_D \pm \frac{1}{2} m_R^c$	$T = T_{now}$ $m_R \approx m_s \approx m_D^2/m_a$ m_a - fixed
$5 \cdot 10^2$ eV 10^2 GeV	m_R $\leq 10^{-3}$ eV m_s $\leq 10^{-3}$ eV m_a 0	3 KeV 10^2 GeV 10^2 GeV	10^{15} GeV 10^{15} GeV 10^{-2} eV
5 eV 1 MeV	m_R $\leq 10^{-6}$ eV m_s $\leq 10^{-6}$ eV m_a 0	30 eV 1 MeV 1 MeV	10^5 GeV 10^5 GeV 10^{-2} eV

Growing Mass, Sterile neutrino DM-candidate

Decreasing Mass, active neutrino

Table 1: Neutrino masses at different temperatures ($\alpha = 1$) for two choices of the Dirac mass m_D. The current active neutrino mass is set $m_a = 10^{-2}$ eV and the quintessence potential scale M is chosen to match the current DE density. All the parameters in the table are defined in the text.

$$\rho_{\varphi\nu,\text{now}} = \rho_{DE} \approx \frac{3}{4} \cdot \frac{3H_0^2}{8\pi G} \approx 31 \cdot 10^{-12}\ \text{eV}^4$$

$$U(\varphi) = \frac{M^{\alpha+4}}{\varphi^{\alpha}}$$

Conclusions:

1. DE: Ratra-Peebles quintessence potential
   Toy Model: DE + Dirac fermion

2. The present Universe is below its critical temperature.
   The first-order phase transition occurs:
   metastable oscillatory →unstable (slow) rolling regime at $z \simeq 4$

4. By choosing M to match the present DE density
   → present neutrino mass
   + redshift where the Universe starts to accelerate $z^* \simeq 0.83$

5. Proposal: Toy model → Extension of the standard model
   (Majorana neutrinos) → Unifying DE-DM-Neutrino
   Further work is needed

THANK YOU !