

My research interests currently center on the spherical functions on symmetric spaces of noncompact type  $G/K$  given by  $\phi_\lambda(e^H) = \int_K e^{(i\lambda - \rho)(\mathcal{H}(e^H k))} dk$  where  $g = k e^{\mathcal{H}(g)} n \in K A N$ .

I have developed some formulae for the spherical functions related to the root systems  $A_{p-1}$  and  $B_p$ . Applications for such formulae include the study of the heat kernel, finding an explicit analytic continuation of  $\phi_\lambda(e^H)$  in  $H$ , the expansion of  $\phi_\lambda(e^H)$  for  $H$  close to 0 (leading to a central limit theorem on symmetric spaces) and an extension of the definition of the spherical functions by allowing the roots (in the root system) to have non integer multiplicities. These topics form an ongoing research project.

In 1999, I started a very fruitful collaboration with Dr. Piotr Graczyk (France). Our research collaboration centers on the “product formula” for spherical functions: as discussed in Helgason’s work, there exists a Weyl-invariant measure  $\mu_{X,Y}$  on  $\mathfrak{a}$  such that

$$\phi_\lambda(e^X) \phi_\lambda(e^Y) = \int_{\mathfrak{a}} \phi_\lambda(e^H) d\mu_{X,Y}(H).$$

We are studying several questions related to this topic. Under which conditions do we have a kernel  $k(H, X, Y)$  such that  $d\mu_{X,Y}(H) = k(H, X, Y) dH$  (*i.e.* absolute continuity of  $\mu_{X,Y}$ )? Assuming that it exists, what are the properties of the kernel  $k(H, X, Y)$ ? Can we find a geometric description of the support of the measure  $\mu_{X,Y}$ ? Is this support convex (the portion that lives in  $\overline{\mathfrak{a}^+}$ )? This problem is reminiscent of the convexity results of Kostant.

These questions have a probabilistic formulation and applications in probability. For example, it allows us to give simple conditions under which the convolution of two  $K$ -bi-invariant measures will be absolutely continuous.