

Dear Author/Editor,

Here are the proofs of your chapter as well as the metadata sheets.

Metadata

- Please carefully proof read the metadata, above all the names and address.
- In case there were no abstracts for this book submitted with the manuscript, the first 10-15 lines of the first paragraph were taken. In case you want to replace these default abstracts, please submit new abstracts with your proof corrections.

Page proofs

- Please check the proofs and mark your corrections either by
 - entering your corrections online
 - or
 - opening the PDF file in Adobe Acrobat and inserting your corrections using the tool "Comment and Markup"
 - or
 - printing the file and marking corrections on hardcopy. Please mark all corrections in dark pen in the text and in the margin at least $\frac{1}{4}$ " (6 mm) from the edge.
- You can upload your annotated PDF file or your corrected printout on our Proofing Website. In case you are not able to scan the printout, send us the corrected pages via fax.
- Please note that any changes at this stage are limited to typographical errors and serious errors of fact.
- If the figures were converted to black and white, please check that the quality of such figures is sufficient and that all references to color in any text discussing the figures is changed accordingly. If the quality of some figures is judged to be insufficient, please send an improved grayscale figure.

Metadata of the chapter that will be visualized online

Book Title	Interdisciplinary Topics in Applied Mathematics, Modeling and Computational Science	
Chapter Title	Monte Carlo Study of the Random Image Area Estimation by Pairwise Comparisons	
Copyright	Springer International Publishing Switzerland 2014	
Corresponding Author	Family name	Koczkodaj
	Particle	
	Given name	W.W.
	Suffix	
	Division	
	Organization	Laurentian University
	Address	Sudbury, ON, Canada
	email	wkoczkodaj@cs.laurentian.ca
Author	Family name	Almowanes
	Particle	
	Given name	A.
	Suffix	
	Division	
	Organization	Laurentian University
	Address	Sudbury, ON, Canada
Author	Family name	Kakiashvili
	Particle	
	Given name	T.
	Suffix	
	Division	
	Organization	Brain Research, Baycrest
	Address	Toronto, ON, Canada
Author	Family name	Duncan
	Particle	
	Given name	G.
	Suffix	
	Division	
	Organization	Laurentian University
	Address	Sudbury, ON, Canada
Abstract	This study presents experimental results of gaining the accuracy of 18.4% when the pairwise comparisons method was used instead of the direct method for area estimation of random images. Random images were produced by deblurring the Gaussian blur applied to randomly generated polygons. Participants were asked to estimate the areas of five random images by using an online questionnaire. Images have been compared to a provided unit of measure and in pairs. Our intensive Internet searches could not find another Monte Carlo experimentation for 2D case conducted in the past.	

Monte Carlo Study of the Random Image Area Estimation by Pairwise Comparisons

W.W. Koczkodaj, A. Almowanes, T. Kakiashvili and G. Duncan

1 **Abstract** This study presents experimental results of gaining the accuracy of 18.4 %
2 when the pairwise comparisons method was used instead of the direct method for
3 area estimation of random images. Random images were produced by deblurring the
4 Gaussian blur applied to randomly generated polygons. Participants were asked to
5 estimate the areas of five random images by using an online questionnaire. Images
6 have been compared to a provided unit of measure and in pairs. Our intensive Internet
7 searches could not find another Monte Carlo experimentation for 2D case conducted
8 in the past.

9 1 Introduction

10 Random images with smooth-looking edges were used in our Monte Carlo study.
11 Such random images that were not too difficult to estimate their area. For it, we
12 used a simple heuristic for generating these placated nice random images based on a
13 modified technique in [9] posted in 2008. In reality, no one can categorically say what
14 a nice image is. However, we can recognize nice images once we see them and more
15 importantly, we can generate them. Smoothing the edges by deblurring help us to
16 generate such images. However, this study is about accuracy, not the random image
17 generation and the “quality” of randomness was not the subject of our investigation.

18 The pairwise comparisons is a useful method especially for processing subjective
19 data. Its main goal is to establish the relative preference of n stimuli in situations
20 where it is impractical to provide estimates for the stimuli [3]. The pairwise compar-
21 isons method can always be used to reach final conclusions elegantly. This method
22 is of considerable importance in situations where direct measurements are impossi-
23 ble to perform. It provides a natural and a powerful tool for decision making. It is
24 a natural approach for processing subjectivity, although objective data can also be
25 processed this way. By common sense, and for any type of comparisons, taking two

W. W. Koczkodaj (✉) · A. Almowanes · G. Duncan
Laurentian University, Sudbury, ON, Canada
e-mail: wkoczkodaj@cs.laurentian.ca

T. Kakishvili
Brain Research, Baycrest, Toronto, ON, Canada

© Springer International Publishing Switzerland 2014

M. G. Cojocaru et al. (eds.), *Interdisciplinary Topics in Applied Mathematics, Modeling and Computational Science*, Springer Proceedings in Mathematics & Statistics 117, DOI 10.1007/978-3-319-12307-3_39

1

criteria or alternatives at a time works better than taking all of them at once. Evidently, handling multiple things at once is more difficult. The pairwise comparisons method is often used to subjectively compare objects. In particular, this method is used to compare objects that are difficult or impossible to measure. For example, there is no defined measure unit for the public satisfaction. The pairwise comparisons method is used for ranking all kinds of preferences and decision making. In some situations, it is the only feasible method where subjectivity is a dominant factor for a decision making.

To perform the random image Monte Carlo accuracy testing of pairwise comparisons, an online questionnaire was implemented and acted as our data collection method. Participants were asked to estimate areas of five images using a provided unit. In addition, they were asked to compare the images in pairs. The average error rate was then calculated for both and compared. The results were encouraging as the gain of accuracy reached 18.4 % when the pairwise comparisons method was used. To our own knowledge and based on an intensive search, this is the first Monte Carlo study for 2D accuracy testing of pairwise comparisons.

2 The Survey Design

Our 2D Monte Carlo experimentation for testing the pairwise comparisons method accuracy is based on using random images. The former 1D experiment in [7] was based on randomly generated bars. In [1], random images were used but of equal area. Participants related the areas of five randomly generated images of equal area. A reference unit area was also displayed along with the images. Respondents' average error when estimating the area using the unit square was 25.75 %. Nevertheless, the error went down to 5.51 % when the images were compared in pairs. It is a much better improvement percentage than the 1D case where bars were used [7]. The experiment demonstrated in [1] is the first 2D statistical experiment showing that the pairwise comparisons method improves accuracy but it was conducted for random images equal in size. In [1], a sample of 179 participated in the study. In the first part of that experiment, they were asked to estimate the area of five randomly generated images of equal areas in units. Of course, respondents were not told that the images were equal in area. The images were presented in an overhead screen and participants took, on average, 10–15 s to estimate the area of each image. In the second part, the images were shown in pairs. Ten pairs were shown and similarly it took 10–15 s to compare each pair. For each pair, participants were asked which image is larger. They also had the option to respond if they believed that a pair was equal.

Generating random images is based on deblurring in [4, 5]. In 2008, an implementation in Photoshop has been posted on the Internet [9]. A special “graphical” type of a questionnaire has been designed, implemented, and programmed in Hypertext Preprocessor (PHP). The questionnaire was posted on a web page for the data collection process. The following section provides a detailed description of the data acquisition.

[AQ1] Remove “special”: It should be: (A “graphical” type of a questionnaire ...)

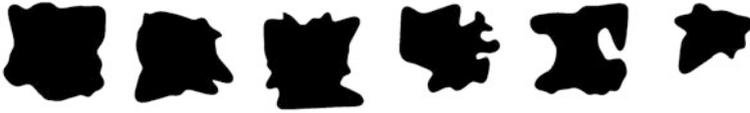


Fig. 1 Randomly generated images with unequal area sizes

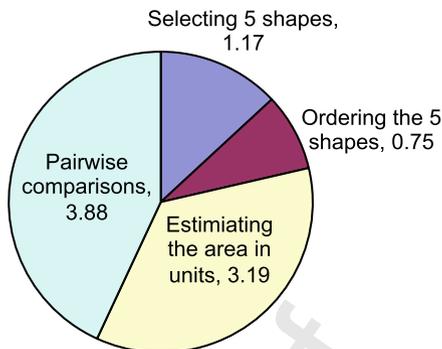
2.1 Data Acquisition Application

There are 93 recorded observations used in this experiment. There was no particular procedure for selecting participants. Only the date, time, and participants' answers were recorded. The email was also recorded only if participants asked for the results to be sent to them when the study will be completed. No Internet Protocols (IPs) or any personal identification were stored. In the first part of the experiment, participants were asked to choose 5 images from a pool of 70 images similar to the images shown in Fig. 1. They were rescaled to a smaller size (63×63) to make all 70 images fit the screen.

Users were asked to put in order the five randomly generated images from the largest to the smallest, where the largest gets the value of 1 and the smallest gets 5. This is to ensure that the user is able to distinguish the visible size difference among the images. In addition, it gives the ability to be consistent in the way the pair of images is displayed on the ten pairwise comparisons screens. The system allows the user to proceed to the area estimation in units page only if the ordering is correct. Otherwise, they would need to select five new images. We decided for the square unit, used in the direct method, to be of size 1600 pixels. That is a 40×40 unit square. The user can only input valid numeric values. If the user inputs an invalid value, an appropriate error message will be shown. If a value is valid and the submit button is clicked, the user will be taken to the next page. In the last part of the experiment, participants were shown two of the five random images side by side (pairwise comparisons). The larger image is always displayed on the left side. There were ten unique pairs that can be formed from the five images. So, ten comparisons were performed.

Polygons are then generated and filled with black and a Gaussian blur is applied to make rough edges smooth. Afterward, a threshold to transform gray pixels to black or white is used. The next step was to scale all 70 images to make them equal in area with $< 0.1\%$ margin at most. The areas are then recorded and saved to a MySQL database for easy access through PHP. We also needed to be sure that the five selected images are displayed to the user in 1–5 ratio from largest to smallest. That is why we performed the previous step of rescaling all images to approximately equal in area images and then applying a new random scale to have the five images in a 1–5 ratio. This can be done by manipulating how the image is displayed in the browser. Next, images are displayed on the ranking screen in no particular order. The user then orders them from largest to smallest.

Fig. 2 A pie chart that shows the average time taken to complete each task in minutes



102 2.2 Computing the Survey Results

103 The collected data have been transformed into a pairwise comparisons matrix M of
104 the size 5 by 5:

$$M = \begin{bmatrix} 1 & m_{12} & \cdots & m_{1n} \\ \frac{1}{m_{12}} & 1 & \cdots & m_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{m_{1n}} & \frac{1}{m_{2n}} & \cdots & 1 \end{bmatrix}.$$

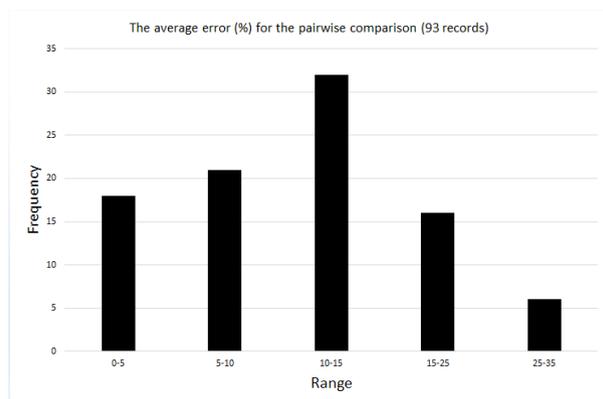
105 We used the theory presented in [6] as the distance-based inconsistency, extended in
106 [2], and finally simplified in [8] as:

$$ii = 1 - \min(x * z/y, y/x/z), \quad (1)$$

107 for a triad (x, y, z) with all strictly positive coordinates.

108 The average error rate when estimating the area of random images in units (direct
109 method), is 30.3 % for the 93 observations. On the other hand, the average error
110 rate is only 11.96 % when the pairwise comparisons method is used, and this can
[AQ2] 111 be seen in Fig. 3. The gain of accuracy here is approximately 18.4 %. The results
112 are highly encouraging. The drop of estimation error, from 30.3 to 11.96 % (see
113 Fig. 4), is even more spectacular than the 1D case reported in [7]. It is evident
114 that the accuracy improves when random images' area estimation using the pairwise
115 comparisons method is enforced.

116 As shown in Fig. 2, the total average time that the participants needed to complete
117 all tasks, is approximately 9 min. Although the average time taken to complete both
118 the direct and pairwise comparisons methods are similar, the accuracy improves
119 dramatically when the pairwise comparisons method is used.



[AQ2]Fig 3 and 4 is now better quality

Fig. 3 Histogram showing the average error when using the pairwise comparisons method

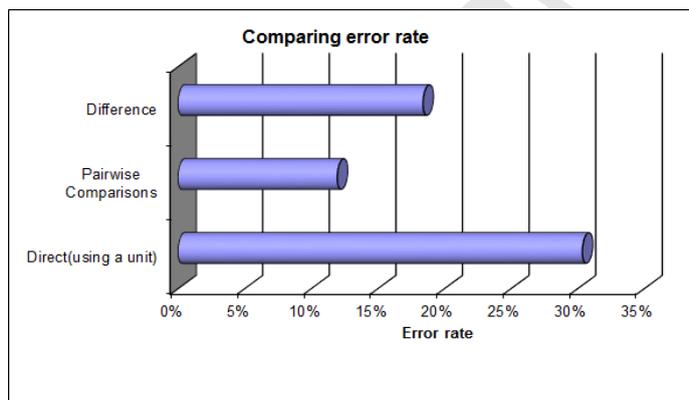


Fig. 4 Comparing the average error rate when using the pairwise comparisons and the direct methods for area estimation of random images

120 3 Conclusion

121 The results of our Monte Carlo experiment strongly favor the pairwise comparisons
 122 method over the direct method. The average error for the pairwise comparisons is
 123 nearly 11.96 versus 30.3 % when the direct method is used. The gain of accuracy,
 124 which is the difference between the errors derived from the direct method and the
 125 pairwise comparisons method, is around 18.4 %. It is even more impressive than the
 126 1D case reported in [7] conducted 18 years ago. It is also worth mentioning that the
 127 average time taken to complete both the direct and pairwise comparisons methods
 128 was close, but the accuracy improves dramatically when the pairwise comparisons
 129 method is used.

130 **Acknowledgement** A. Almowanes would like to acknowledge the endeavors of the sponsor, the
131 Kingdom of Saudi Arabia, Ministry of Higher Education.

132 **References**

- 133 1. Adamic, P., Kakiashvili, T., Koczkodaj, W.W., Babiy, V., Janicki, R., Tadeusiewicz, R.: Pairwise
134 comparisons and visual perceptions of equal area polygons. *Percept. Motor Skills* **108**(1):37–42
135 (2009)
- 136 2. Duszak, Z., Koczkodaj, W.W.: Generalization of a new definition of consistency for pairwise
137 comparisons. *Inf. Process. Lett.* **52**(5):273–276 (1994)
- 138 3. Herman, M., Koczkodaj, W.W.: A Monte Carlo study of pairwise comparison. *Inf. Process. Lett.*
139 **57**(1):25–29 (1996)
- 140 4. Hummel, R.A., Kimia, B., Zucker, S.W.: Deblurring Gaussian blur. *Comput. Vision Gr. Image*
141 *Process.* **38**(1):60–80 (1987)
- 142 5. Immerkaer, J.: Use of blur-space for deblurring and edge-preserving noise smoothing. *IEEE*
143 *Trans. Image Process.* **10**(6):837–840 (2001)
- 144 6. Koczkodaj, W.W.: A new definition of consistency of pairwise comparisons. *Math. Comput.*
145 *Model.* **18**(7):79–84 (1994)
- 146 7. Koczkodaj, W.W.: Statistically accurate evidence of improved error rate by pairwise compar-
147 isons. *Percept. Motor Skills* (1996). doi:10.2466/pms.1996.82.1.43
- 148 8. Koczkodaj, W.W., Szwarc, R.: On axiomatization of inconsistency indicators in pairwise
149 comparisons. *CoRR abs/1307.6272* (2013)
- 150 9. RandomMetropolis: Tutorial—smooth edges in Photoshop! Available via YouTube. [http://www.](http://www.youtube.com/watch?v=m9hicX0gyXI)
151 [youtube.com/watch?v=m9hicX0gyXI](http://www.youtube.com/watch?v=m9hicX0gyXI) (2008) Accessed 1 Feb 2013

Chapter 39: Author Query

AQ1. Please check whether the edits made in the sentence "A special..." retain your intended sense.

AQ2. Figures 3 and 4 are of low quality. Please provide better quality images.