On Saaty's and Koczkodaj's inconsistencies of pairwise comparison matrices

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Abstract

The aim of the paper is to obtain some theoretical and numerical properties of Saaty's and **Koczkodaj**'s inconsistencies of pairwise comparison matrices (PRM). In the case of 3 x 3 PRM, a differentiable one-to-one correspondence is given between Saaty's inconsistency ratio and **Koczkodaj**'s inconsistency index based on the elements of PRM. In order to make a comparison of Saaty's and **Koczkodaj**'s inconsistencies for 4 x 4 pairwise comparison matrices, the average value of the maximal eigenvalues of randomly generated n x n PRM is formulated, the elements a(ij) (i < j) of which were randomly chosen from the ratio scale

1/M, 1/M - 1,..., 1/2, 1, 2, ..., M -1, M,

equal probability 1/(2M -1) and a(ji) is defined as 1/a(ij) . By statistical analysis, the empirical distributions of the maximal eigenvalues of the PRM depending on the dimension number are obtained. As the dimension number increases, the shape of distributions gets similar to that of the normal ones. Finally, the inconsistency of asymmetry is dealt with, showing a different type of inconsistency.

An R package for analyzing and modeling ranking data

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Abstract

Background: In medical informatics, psychology, market research and many other fields, researchers often need to analyze and model ranking data. However, there is no statistical software that provides tools for the comprehensive analysis of ranking data. Here, we present pmr, an R package for analyzing and modeling ranking data with a bundle of tools. The pmr package enables descriptive statistics (mean rank, pairwise frequencies, and marginal matrix), Analytic Hierarchy Process models (with Saaty's and **Koczkodaj**'s inconsistencies), probability models (Luce model, distance-based model, and rank-ordered logit model), and the visualization of ranking data with multidimensional preference analysis.

Results: Examples of the use of package pmr are given using a real ranking dataset from medical informatics, in which 566 Hong Kong physicians ranked the top five incentives (1: competitive pressures; 2: increased savings; 3: government regulation; 4: improved efficiency; 5: improved quality care; 6: patient demand; 7: financial incentives) to the computerization of clinical practice. The mean rank showed that item 4 is the most preferred item and item 3 is the least preferred item, and significance difference was found between physicians' preferences with respect to their monthly income. A multidimensional preference analysis identified two dimensions that explain 42% of the total variance. The first can be interpreted as the overall preference of the seven items (labeled as "internal/external"), and the second dimension can be interpreted as their overall variance of (labeled as "push/pull factors"). Various statistical models were fitted, and the best were found to be weighted distance-based models with Spearman's footrule distance.

Conclusions: In this paper, we presented the R package pmr, the first package for analyzing and modeling ranking data. The package provides insight to users through descriptive statistics of ranking data. Users can also visualize ranking data by applying a thought multidimensional preference analysis. Various probability models for ranking data are also included, allowing users to choose that which is most suitable to their specific situations.

Generalization of the RCGM and LSLR pairwise comparison methods
By: Limayem, F; Yannou, B
COMPUTERS & MATHEMATICS WITH APPLICATIONS Volume: 48 Issue: 3-4 Pages: 539-548 Published: AUG 2004
Abstract
Pairwise comparison methods are convenient procedures for predicting a sound weight vector from a set of relative comparisons between elements to be weighted. Several pair-wise comparison methods exist. After a brief presentation of the least squares logarithmic regression (LSLR) method of de Graan [1] and Lootsma [2] and the recent row and column geometric mean (RCGM) of **Koczkodaj** and Orlowski [3], this paper proposes a common mathematical formulation for these two approaches. This common formulation leads to two generalized methods. The GLSLR is now able to process nonreciprocal comparison matrices, and the GRCGM is extended to several decision makers expressing different opinions per pairwise comparison. It also results in an explicit formulation of the weights that generalizes **Koczkodaj** and Orlowski's formulation of the closest consistent comparison matrix.

On reducing inconsistency of pairwise comparison matrices below an acceptance threshold
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(Submitted on 4 Nov 2013)
A recent work of the authors on the analysis of pairwise comparison matrices that can be made consistent by the modification of a few elements is continued and extended. Inconsistency indices are defined for indicating the overall quality of a pairwise comparison matrix. It is expected that serious contradictions in the matrix imply high inconsistency and vice versa. However, in the 35-year history of the applications of pairwise comparison matrices, only one of the indices, namely CR proposed by Saaty, has been associated to a general level of acceptance, by the well known ten percent rule. In the paper, we consider a wide class of inconsistency indices, including CR, CM proposed by **Koczkodaj** and Duszak and CI by Pel\'aez and Lamata. Assume that a threshold of acceptable inconsistency is given (for CR it can be 0.1). The aim is to find the minimal number of matrix elements, the appropriate modification of which makes the matrix acceptable. On the other hand, given the maximal number of modifiable matrix elements, the aim is to find the minimal level of inconsistency that can be achieved. In both cases the solution is derived from a nonlinear mixed-integer optimization problem. Results are applicable in decision support systems that allow real time interaction with the decision maker in order to review pairwise comparison matrices.
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http://arxiv.org/abs/1311.0748

Alessio Ishizaka and Ashraf Labib
Review of the main developments in the analytic hierarchy process, Expert Systems with Applications, 38(11), 14336-14345, 2011:

> If mathematical evidences testify clearly for the geometric mean over the eigenvalue method, there is no clear differences between these two methods when simulations are applied (Budescu, Zwick, & Rapoport, 1986; Cho & Wedley, 2004; Golany & Kress, 1993; Herman & **Koczkodaj**, 1996; Ishizaka & Lusti, 2006; Jones & Mardle, 2004; Mikhailov & Singh, 1999), apart from special cases (Bajwa, Choo, & Wedley, 2008). Perhaps in the light of this lack of practical evidence, Saaty's group has always supported the eigenvalue method (Harker & Vargas, 1987; T. Saaty, 2003; T. Saaty & Hu, 1998; T. Saaty & Vargas, 1984a, 1984b).
> [Herman, M., & Koczkodaj, W. (1996). A Monte Carlo Study of Pairwise Comparison. *Information Processing Letters 57*, 25-29.]

In a monograph:

> Statistical Modeling for Management, Graeme D. Hutcheson and Luiz Moutinho, 2008

'Koczkodaj' is present in:

> the habilitation thesis of :
>
> - ❏ Gabriele Kotsis (1999), Univeristy of Vienna, Austria
> - ❏ Bernard Yannou (2001), France
>
> the PhD dissertation of:
>
> - ❏ Mohammad Hadi Soudkhah, McMaster University (2014), Canada
> - ❏ Yun Zhai, McMaster University (2010), Canada
> - ❏ Jaesung Sim, University of North Texas (2003), USA
> - ❏ Abigail R. Clarke-Sather, *Michigan Technological University*, USA
> - ❏ Nor Badrul Anuar Jumaat (2012), Plymouth University, UK
> - ❏ Rita Markovits-Somogyi , Budapest University of Technology and Economics , Hungary
> - ❏ Aleksander Øhrn, Norwegian University of Science and Technology, Norway
> - ❏ Sándor Bozóki, Corvinus University of Budapest, Hungary,
> - ❏ Matteo Brunelli, Abo Akademi University (2011), Finland

- F. Limayem, Ecole Centrale Paris (2001), France

[The search has been discontinued due to lack of time]