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PAIRWISE COMPARISONS AND VISUAL PERCEPTIONS OF EQUAL AREA POLYGONS [1, 2]

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Summary.—The number of studies related to visual perception has been plentiful in recent years. Participants rated the areas of five randomly generated shapes of equal area, using a reference unit area that was displayed together with the shapes. Respondents were 179 university students from Canada and Poland. The average error estimated by respondents using the unit square was 25.75%. The error was substantially decreased to 5.51% when the shapes were compared to one another in pairs. This gain of 20.24% for this two-dimensional experiment was substantially better than the 11.78% gain reported in the previous one-dimensional experiments. This is the first statistically sound two-dimensional experiment demonstrating that pairwise comparisons improve accuracy.

An experiment was designed to assess if using pairwise comparisons, a method first mentioned in de Caritat and de Condorcet (1972) and in Fechner (1860), yields more precision in estimation of the areas and equality of randomly generated shapes. Although the experiment is related to the one-dimensional case reported in Koczkodaj (1996), the method is not as general as in Koczkodaj (1996), since the shapes are all of equal area. A three-dimensional perception study is forthcoming, more in keeping with the modern perception literature that has emphasized three-dimensional acuity experiments (cf. Todd, 2004 or Fleming, Torralba, & Adelson, 2004).

At the current stage of pairwise comparisons theory, there is no possibility of proving (or disproving) by analytical means which method is superior. The necessity of using computer technology may explain why the problem of accuracy had not been properly addressed in the 1950s and 1960s when most of the theoretical work on the pairwise comparisons method transpired. As such, the main purpose of this experiment is to compare the accuracy of area assessments based on the pairwise comparisons method with that of the direct method, which is also referred to as the *by-eye estimation* method for obvious reasons.

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Method

Sample

For this study, 179 university students (45% women) estimated sizes of geometric shapes. The students were from Laurentian University in Canada (40%) and from the AGH University of Technology in Poland (60%). Since they were undergraduate students, the vast majority of respondents would have been approximately 20 years of age, but exact ages were not recorded.

Measures

In the first part of the experiment, respondents in intact classes were expected to estimate the area of a given shape by using the square unit area as shown in Fig. 1. They were not aware that each randomly generated shape had exactly the same number of area units, which was exactly ten. There were five such comparisons made, each lasting approximately 10 to 15 seconds each. Shapes were presented on an overhead screen.

Fig. 1. Random shapes used in the experiment

In the second part of the experiment, students were shown two random shapes side by side, selected from the same shapes shown in Fig. 1 (again, each for approximately 10 to 15 seconds). There were ten such comparisons, which is the total number of distinct pairs that could be selected from the group of five objects. Respondents were asked, for each of the ten pairs of shapes, to select which shape (L for left or R for right) they felt had a larger area (or E for equal). They estimated by what factor the larger shape was bigger than the smaller shape, for example, 1.4 would mean that the larger shape had a 40% larger area than the smaller shape. Respondents who answered the questionnaire incorrectly were omitted from the analysis. A total of 179 students were included in the study, resulting in 895 responses for the first part and 1,790 responses for the second part.

Stimuli

In this experiment, five two-dimensional random shapes of equal area were generated and then presented to the respondents for comparisons, without revealing the fact that all shapes were of equal area, although the rating scale for area estimation allowed selection of "equal" as a response. Generation of acceptable random shapes is not a trivial task. First, the random shapes should not be too tricky to estimate (for example, a porcupine or a sun with many rays). On the other hand, the random shapes should not be overly simplistic, such as randomly generated squares or circles. The totally random polygon is not an optimal choice for area estimation, and therefore a more comprehensive approach was used for constructing the polygons for the experiment. To this end, a *convex-bottom* method was employed that used polygonizations of point sets in the plane, as described in Deneen and Shute (1988). This method is as random as any other method. The sorting of vertices on the x coordinate is also relatively simple to implement. The shapes were subsequently scrutinized by a scatter diagram with the data points connected by lines.

Procedure

All of the random shapes were randomly generated polygons with a randomly chosen number of vertices, between three and eight. The random generation of all numbers, including coordinates x and y, was done using Microsoft Excel Version 11.0 from an interval of [0,500] for use in a JAVA polygon-drawing program. A randomly generated polygon is composed of a union of triangles and sometimes quadrilaterals.

After the data points were generated and verified, a JAVA program was used to draw the polygons that were then saved in JPEG format. The image was further processed by the GNU Image Manipulation Program (or GIMP) system. Gaussian blur and thresholding were then applied to produce a rounded-corner effect by employing a color cutoff procedure. All shapes were resized to approximately 60,000 pixels (all within 50 pixels of 60,000; the histogram function in GIMP was set to 254 to ensure this) and were saved for the purposes of exporting them to Microsoft PowerPoint Version 11.0. Each of the five shapes was then opened (using an aspect ratio of 100%) and placed in a single file where they could then be grouped in a uniform manner before cutting and pasting them into a single document. The cutting area was always the same each time the images were pasted into PowerPoint, where they were not rescaled. This method for transferring the images was consistently employed. The resulting five shapes that were used for the actual experiment are depicted in Fig. 1.

Analyses

In the pairwise comparisons method, stimuli (shapes in this case) are presented in pairs to respondents. The matrix of pairwise comparisons $\mathbf{A} = [a_{ij}]$ represents the ratio estimations between pairs of shapes (S_i versus S_j, for all $i, j = 1, 2, \ldots n, i < j$). Saaty proposed the eigenvector solution of $\mathbf{A}\gamma = \lambda\gamma$ and proved in Saaty (1977) that a solution always exists if the consistency (or transitivity) condition $a_{ij}a_{jk} = a_{ik}$ for $i, j, k = 1, \ldots, n$ is satisfied.

Unfortunately, in real life and also in this experiment, inconsistency of estimations is the rule; hence, the eigenvalue solution is only an approximation which converges to the simpler geometric mean solution. More details about the problem of inconsistent assessments and definitions of inconsistency can be found, for example, in Bozsóki and Rapcsák (2007), Saaty (1977), and Koczkodaj (1993).

The relative comparison coefficients are entered into a matrix $\mathbf{A} = [a_{ij}]$ for shapes $S_1, S_2, \ldots, S_n$ and are expected to satisfy the equality condition $a_{ii} = 1$ (comparing the same shape with itself should yield 1 on the main diagonal) and $a_{ij} = 1/a_{ji}$, expressing the simple fact that the coefficient of S_i to S_j should be the reciprocal of S_j to S_i. The first constraint is related to comparing a given attribute with itself. The second constraint is a consequence of the obvious fact that $x/y = 1/(y/x)$ for $x, y \neq 0$.

The exact areas of the randomly generated shapes were used to compute the error for both methods as follows. Assuming that a vector $v = [v_1, v_2, \ldots, v_n]$ of the exact shape areas is approximated by the vector $w = [w_1, w_2, \ldots, w_n]$, then the error of this approximation can be defined by the formula

$$d([w_1, w_2, \ldots w_n], [v_1, v_2, \ldots v_n]) = \sum_{i=1}^{n} \frac{|w_i - v_i|}{v_i} \quad .$$

In this experiment, $v_i = 10$ for all i, and the right-hand side can be interpreted as the sum of the absolute values of the relative differences between the generated lengths and the estimated lengths for each observation. The statistical difference between mean errors for each method, the group mean error, and the hypothesis that the estimation error of the pairwise comparisons method is substantially smaller than that of the direct rating method were all examined (see Table 1).

TABLE 1
STATISTICAL RESULTS FROM EXPERIMENT: PERCENT

Estimation Method	Shape 1	Shape 2	Shape 3	Shape 4	Shape 5	All
Direct						
Error	25.12	26.05	29.19	25.38	23.01	25.75
M	9.89	10.12	8.51	9.75	9.61	9.58
Mdn	10.00	10.00	8.00	9.50	9.00	9.30
SD	3.21	3.30	3.23	3.06	2.92	3.14
Pairwise						
Error	6.43	5.23	5.17	6.03	4.67	5.51
M	9.93	10.07	9.94	10.19	9.88	10.00
Mdn	9.99	10.00	10.00	10.00	9.98	9.99
SD	1.02	0.84	0.79	1.27	0.88	0.96

Note.—Estimates of areas for all figures differed significantly between methods, $p < .001$.

Results

For the randomly generated shapes of equal areas, the estimation errors for all of the shapes were similar under the two methods, so the average effect across all shapes was combined. By the direct method, the average error was 25.75%. This was larger than the 15.42% error reported in Koczkodaj (1996) for the one-dimensional experiment. The average estimation error for the pairwise comparisons method was only 5.51%, which was also more sizable than the 3.64% error observed in the one-dimensional experiment. On an intuitive level this should be expected, since two-dimensional estimation is, in general, more complicated than one-dimensional estimation. The gain of accuracy, expressed as the difference between the errors derived from the direct method and pairwise-comparisons method, was 20.24% for two-dimensional and 11.78% for one-dimensional stimuli.

The difference between the direct method and the pairwise-comparisons method is not an artifact. However, to provide evidence that this claim is indeed statistically significant, two sample (paired) t tests were calculated. The two samples were the error rates for each method, the pairwise and the direct methods, respectively. As can be seen in Table 1, all tests were significant at $p < .001$.

Discussion

Using random shapes of identical area avoided previous problems associated with subjective judgment of size. The gain in accuracy, expressed as the difference between the errors derived from the direct method and pairwise-comparisons method, was shown to be 20.24% for two-dimensional stimuli. For one-dimensional stimuli, the gain in accuracy with the latter method was only 11.78%. This is a substantial improvement. Although one cannot judge a trend using only two points, the authors expect that relative error rates will improve with the addition of a third dimension for stimuli.

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